Correlation functions of integrable higher spin XXZ chain in the massive regime

Correlation Functions of Quantum Integrable Models

Friday 9 September 2011

Kohei Motegi

(Okayama Institute for Quantum Physics)

joint work with

Tetsuo Deguchi(Ochanomizu University)

and

Jun Sato(Ochanomizu University)

paper in preparation

correlation functions

spin-1/2 XXZ chain

multiple integral representation

- *q*-vertex operator

  Kyoto(Jimbo, Miwa et al)

- algebraic Bethe ansatz

  Lyon(Kitanine, Maillet, Slavnov Terras et al)

  finite temperature

  Wuppertal(Goehmann, Kluemper et al)

correlation functions
extension to higher spin

- Kitanine
- Maillet, Castro-Alvaredo
- Deguchi, Matsui, Sato
- Goehmann, Seel, Suzuki

this talk
higher spin XXZ chain in the massive regime
one point functions

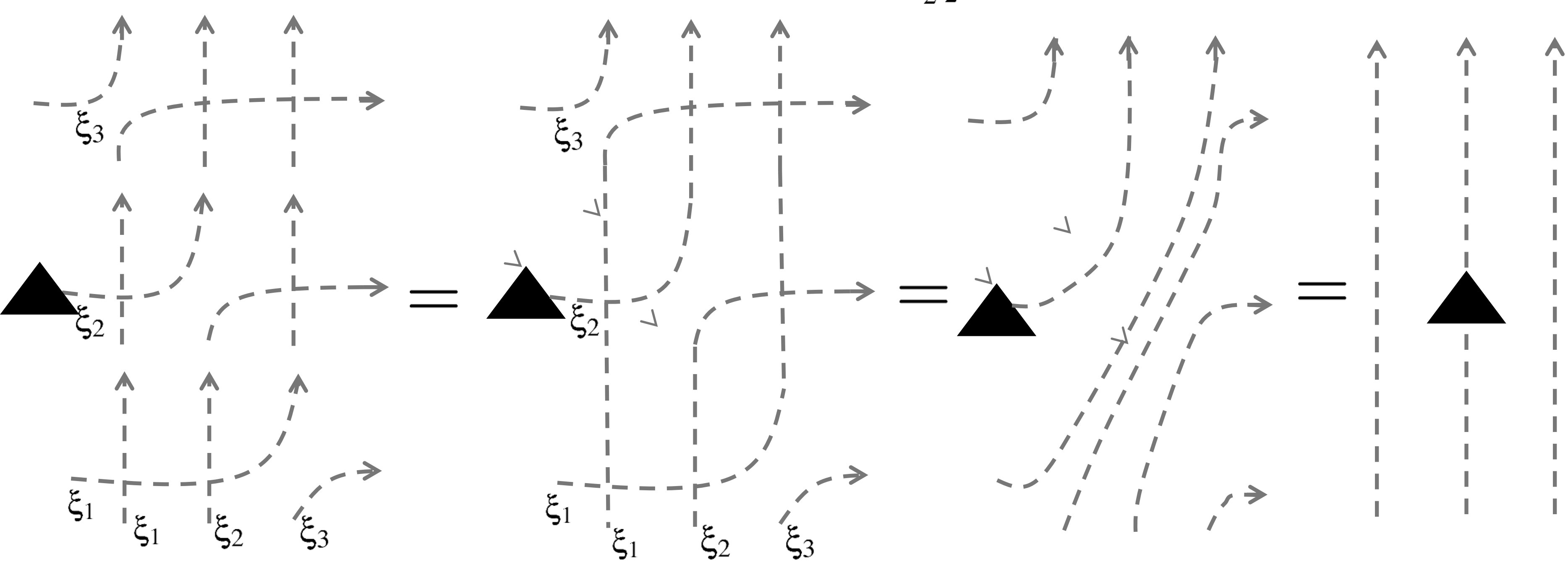
Procedure of deriving the multiple integral representation

spin-1/2 XXZ chain

1. Express correlation functions in terms of monodromy operators($ABCD$ operators)

($T=0$: express local operators in terms of $ABCD$ operators)

$$T(\xi_3)A(\xi_2)T(\xi_1) = |1/2\rangle_2 {}_2\langle 1/2|$$



Procedure of deriving the multiple integral representation

spin-1/2 XXZ chain

1. Express correlation functions in terms of monodromy operators($ABCD$ operators)

($T = 0$: express local operators in terms of $ABCD$ operators)

2. Deform into a form such that the scalar product (Slavnov) formula can be applied

$\downarrow$

mutiple sum representation

Procedure of deriving the multiple integral representation

spin-1/2 XXZ chain

1. Express correlation functions in terms of monodromy operators($ABCD$ operators)

($T=0$: express local operators in terms of $ABCD$ operators)

2. Deform into a form such that the scalar product (Slavnov) formula can be applied

↓

mutiple sum representation

3. Take the thermodynamic limit

↓

multiple integral representation

integrable higher spin XXZ chain

Hamiltonian

$$\Delta = (q + q^{-1})/2, q = \exp(-\eta), \eta > 0$$

$$\frac{(q^{2s} - q^{-2s})}{4s} H = \sum_{j=1}^{L} h_{j,j+1}$$

$$h = \sum_{m_1, m_2 = 0}^{2s} \sum_{n=-\min(m_1, 2s-m_2)}^{\min(m_2, 2s-m_1)} \beta^n_{m_1,m_2} |2s, 2s - 2m_1 - 2n\rangle\langle 2s, 2s - 2m_1| \otimes |2s, 2s - 2m_2 + 2n\rangle\langle 2s, 2s - 2m_2|.$$

$\{(n, m_1, m_2) \mid n \geq 0, m_2 \leq 2s - m_1\},$

$$\beta^n_{m_1,m_2} = \frac{[2s]_q(-1)^{n-1}}{2s[n]_q}\left(\frac{[m_1+n]_q![m_2]_q![2s-m_1-n]_q![2s-m_2]_q!}{[m_1]_q![m_2-n]_q![2s-m_1]_q![2s-m_2+n]_q!}\right)^{1/2}, \ n > 0$$

$$\beta^0_{m_1,m_2} = -\frac{1}{4s}\sum_{l=0}^{m_1-1}\frac{[2s]_q[4s-2l]_q}{[2s-l]_q^2} - \frac{1}{4s}\sum_{l=0}^{m_2-1}\frac{[2s]_q[4s-2l]_q}{[2s-l]_q^2}.$$

$\{(n, m_1, m_2) \mid n \geq 0, m_2 > 2s - m_1\},$

$$\beta^n_{m_1,m_2} = \beta^n_{2s-m_1-n, 2s-m_2+n}.$$

$\{(n, m_1, m_2) \mid n < 0\},$

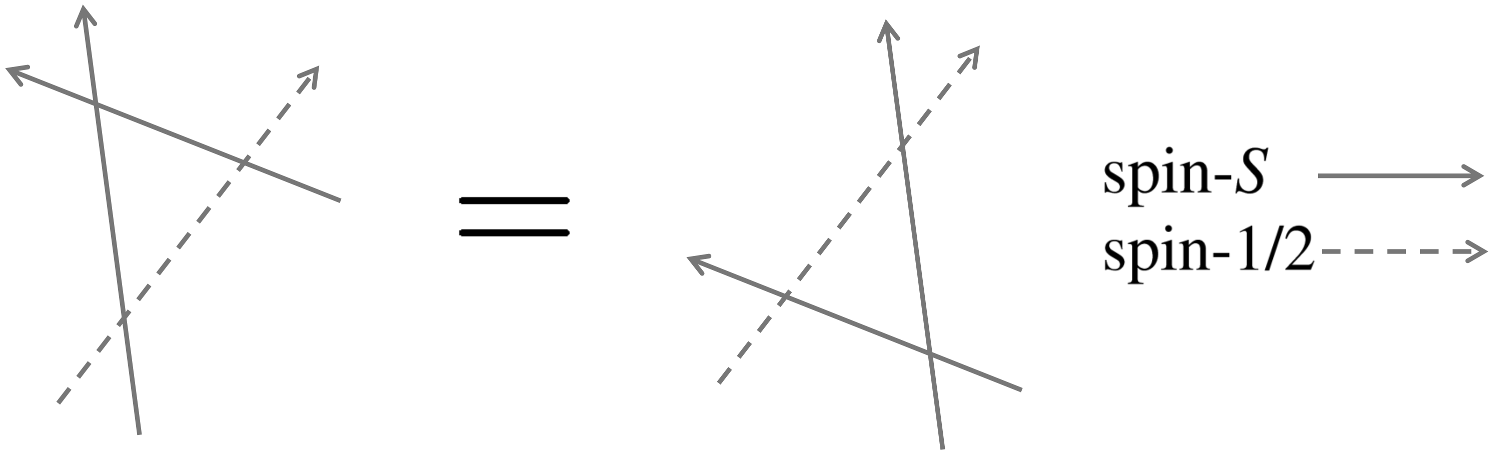
$$\beta^n_{m_1,m_2} = \beta^{-n}_{m_1+n, m_2-n}.$$

($q = 1$: Crampe, Ragoucy, Alonzi)

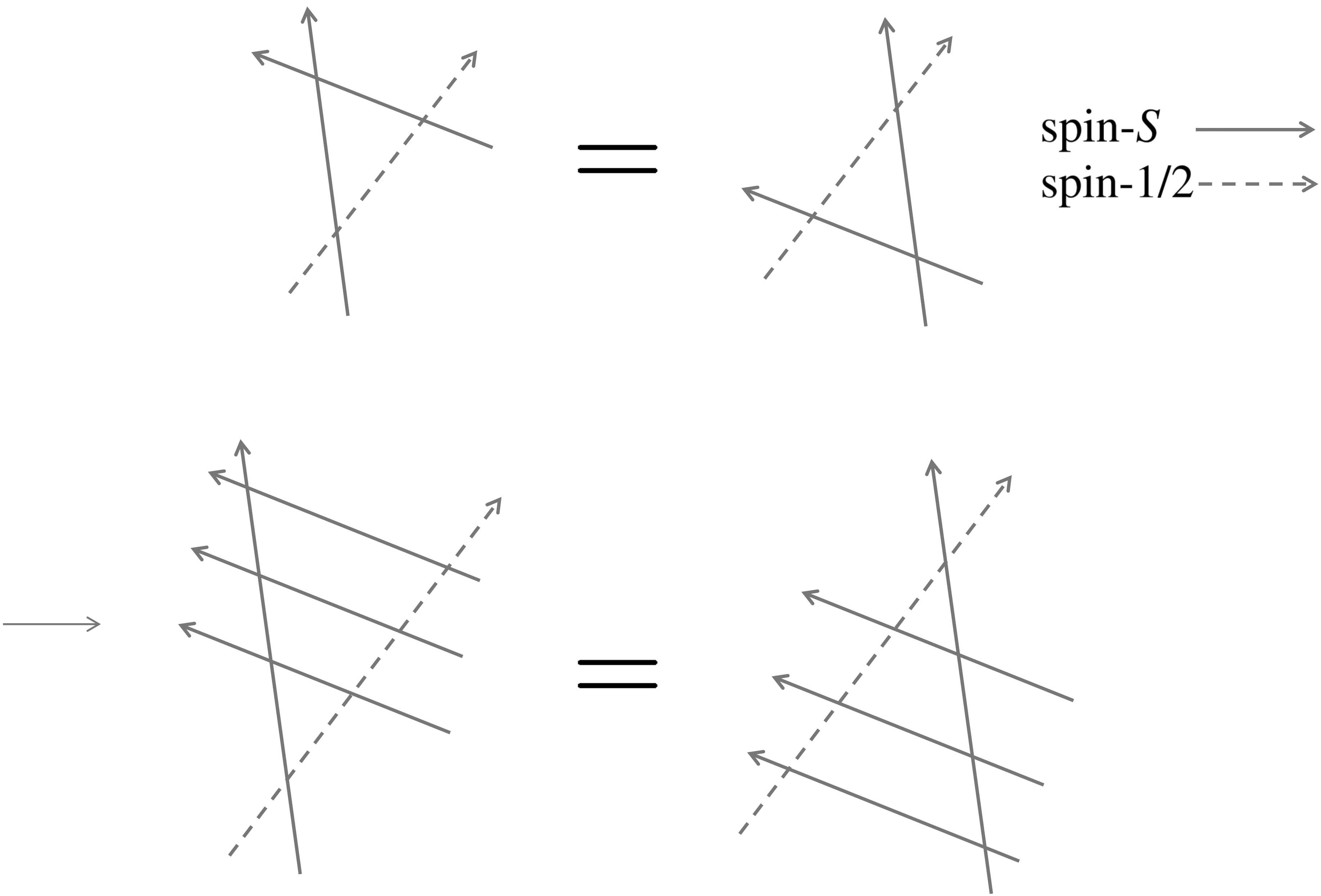
correlation functions

- Babujan's argument
- Fusion

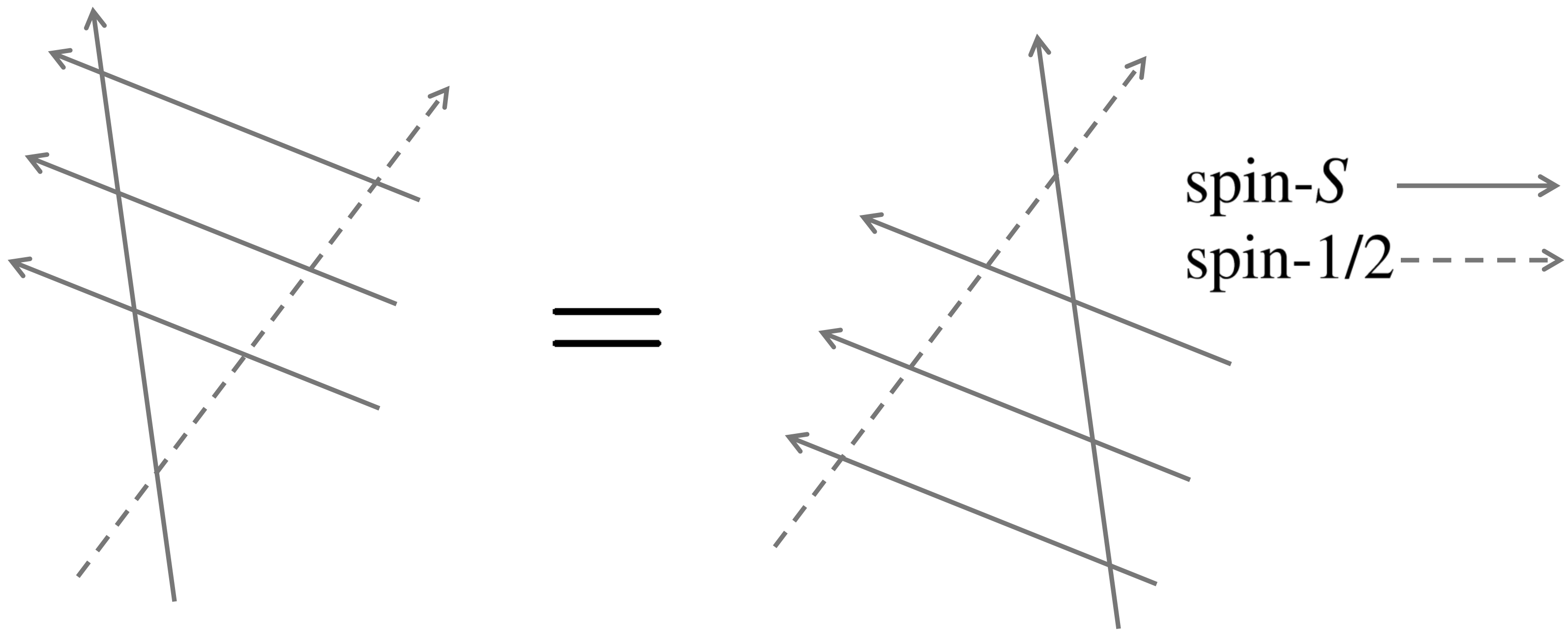
Babujan's argument

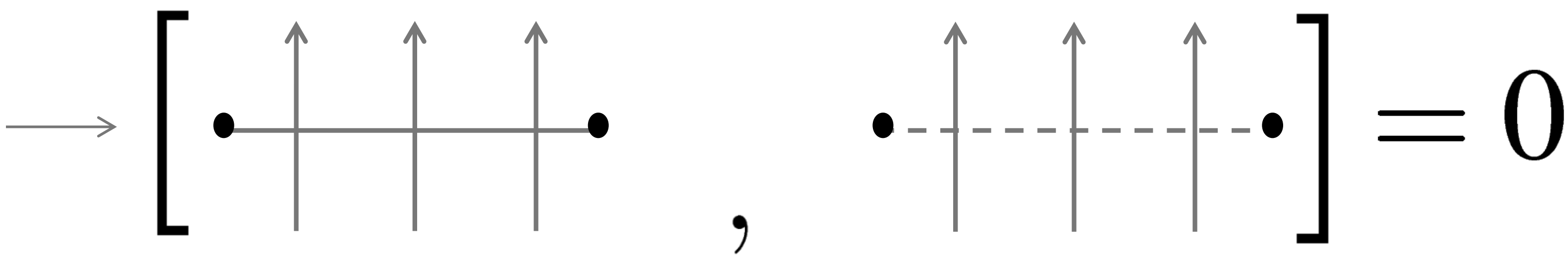


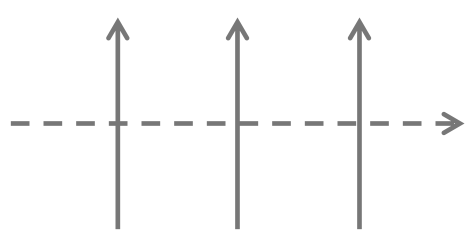
Babujan’s argument



Babujan’s argument





→ can use  ($ABCD$ operators) instead

Procedure of deriving the multiple integral representation

higher spin XXZ chain

1. Express correlation functions in terms of monodromy operators(*ABCD* operators)

Reduce elementary operators of higher spins to elementary operators of spin-1/2

2. Deform into a form such that the scalar product (Slavnov) formula can be applied

↓

mutiple sum representation

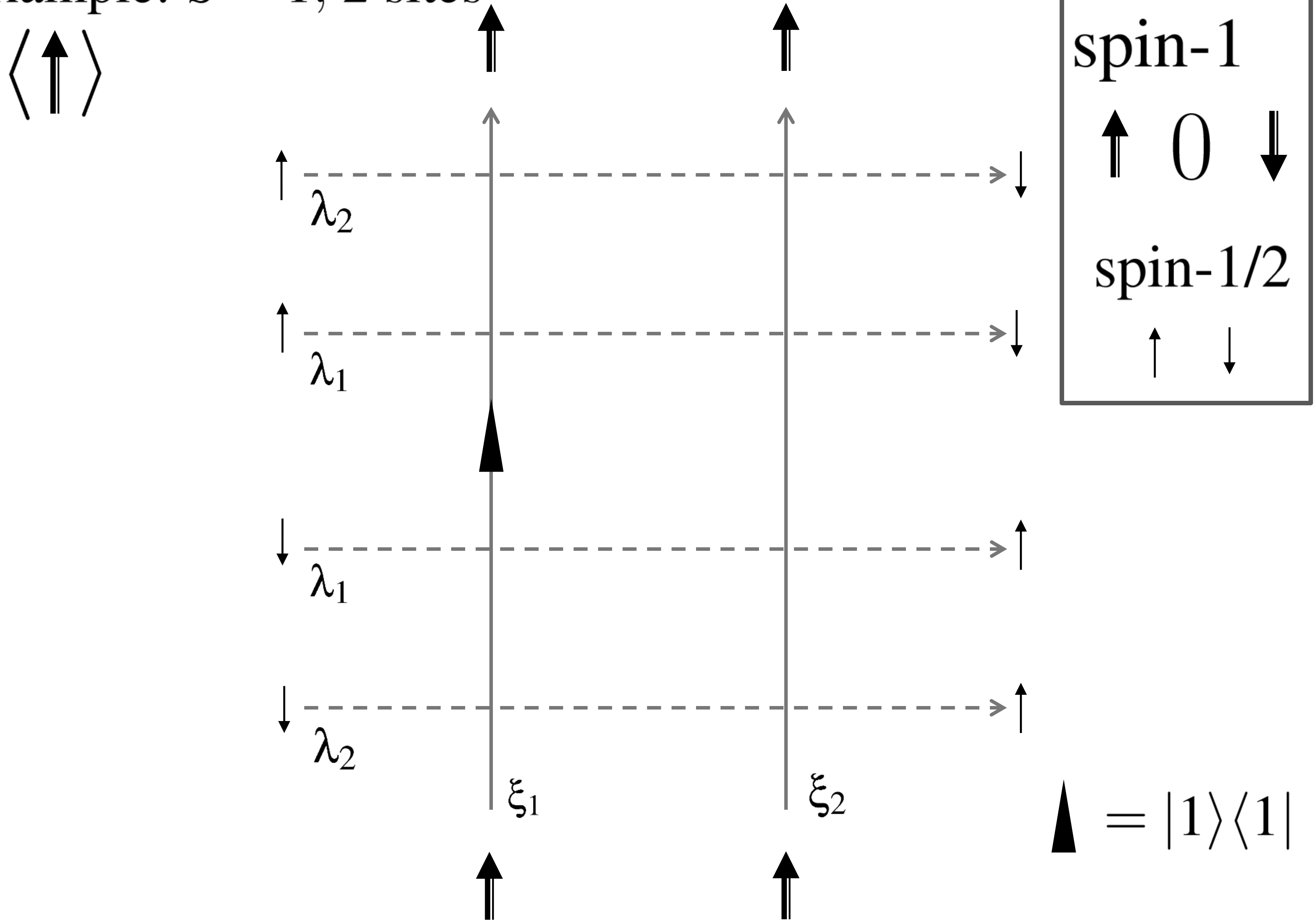
3. Take the thermodynamic limit

Properties of the ground state(Bethe roots)

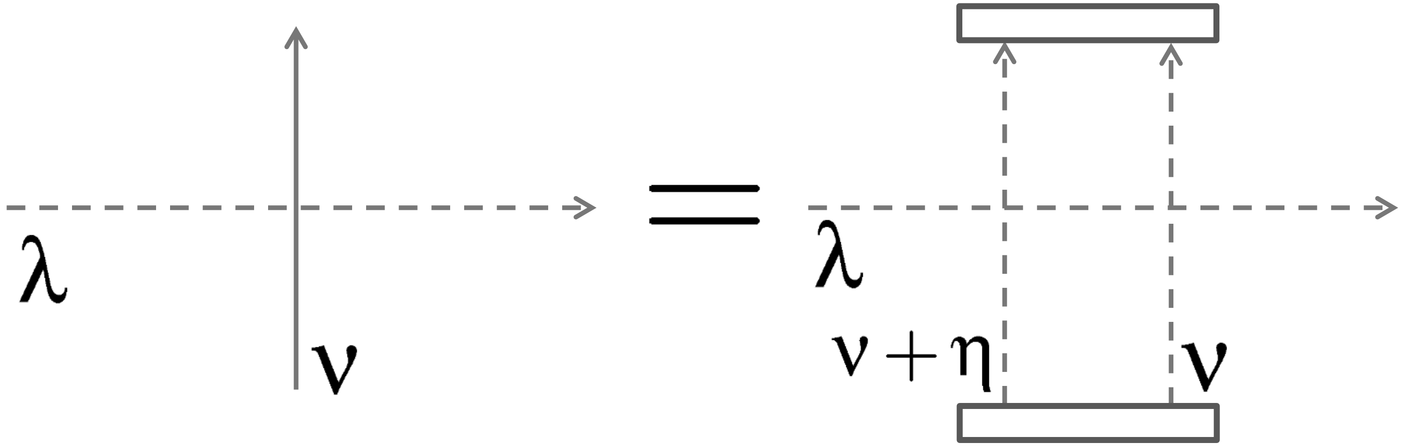
↓

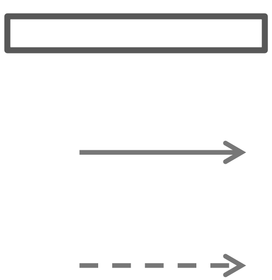
multiple integral representation

Example: $S = 1$, 2 sites



Fusion($S = 1$)



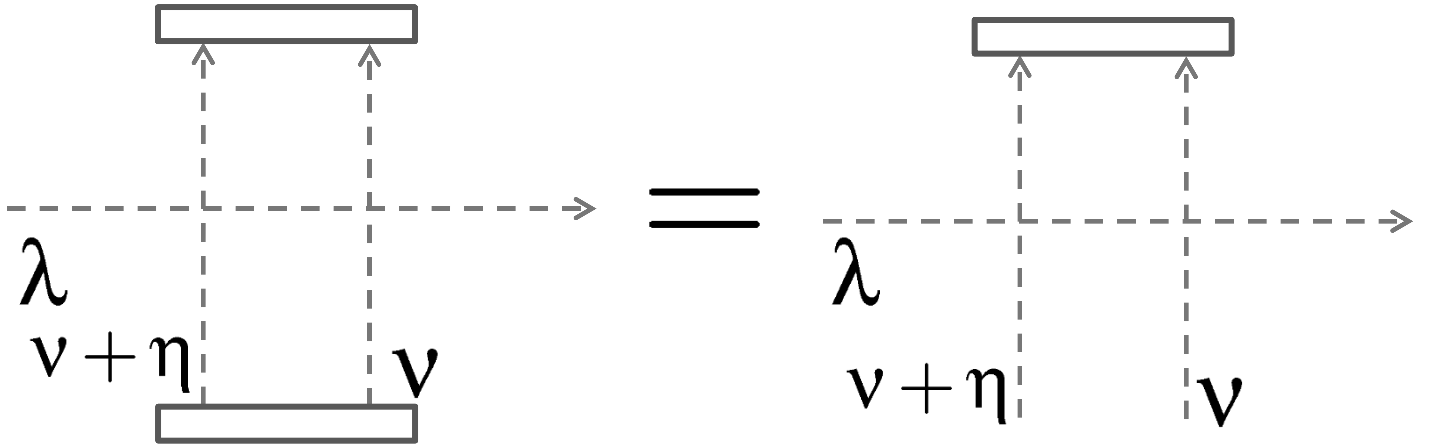


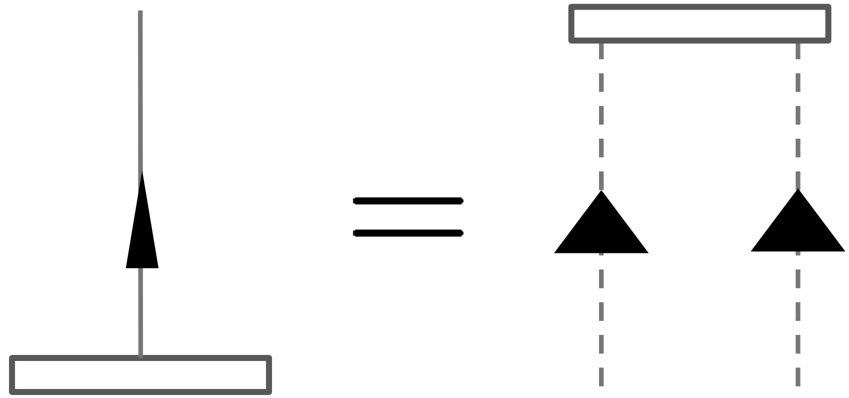
projection operator

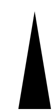
spin-S

spin-1/2

Relations



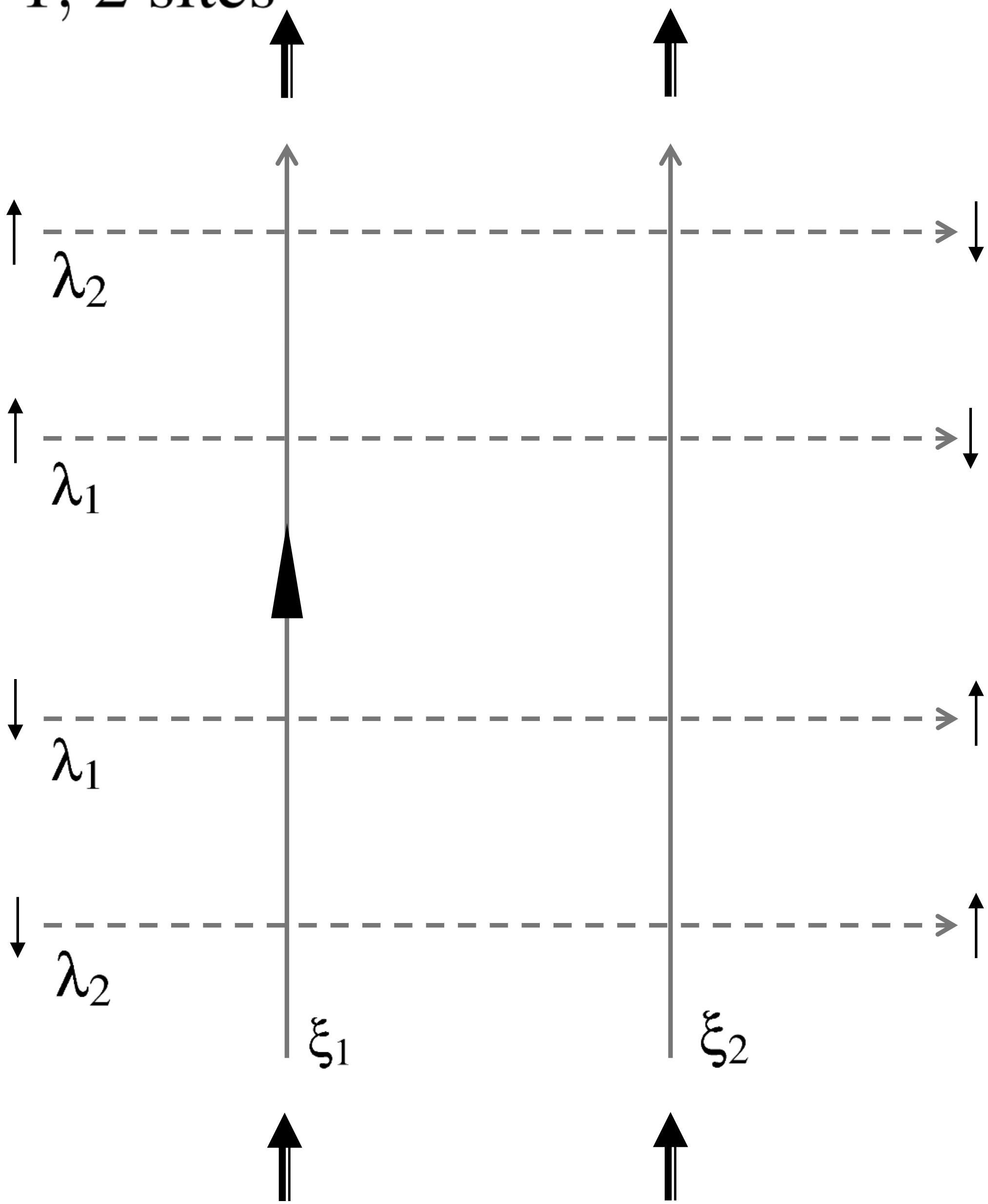


 $= |1\rangle\langle 1|$

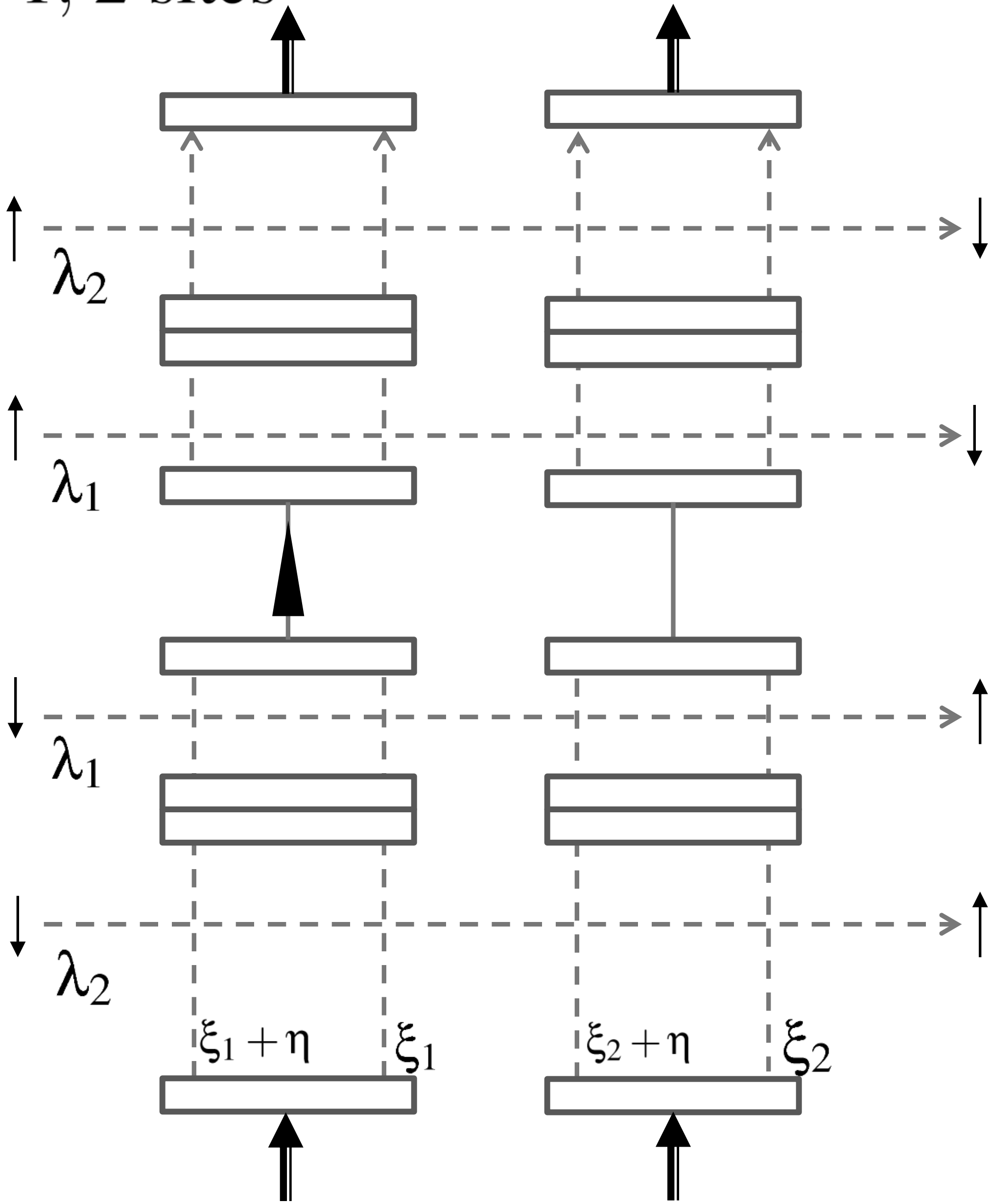
 $= |1/2\rangle\langle 1/2|$

etc.

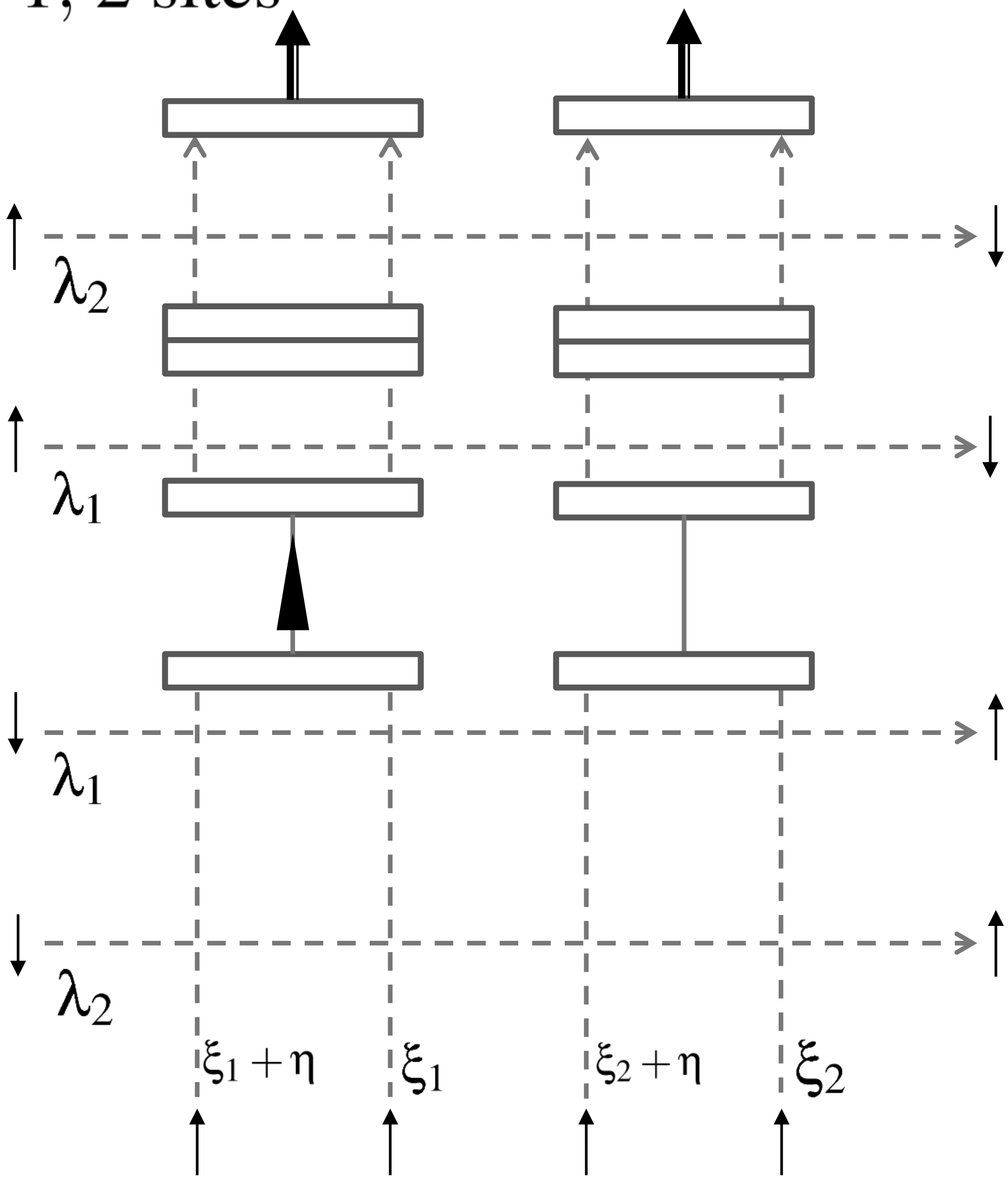
Example: $S = 1$, 2 sites



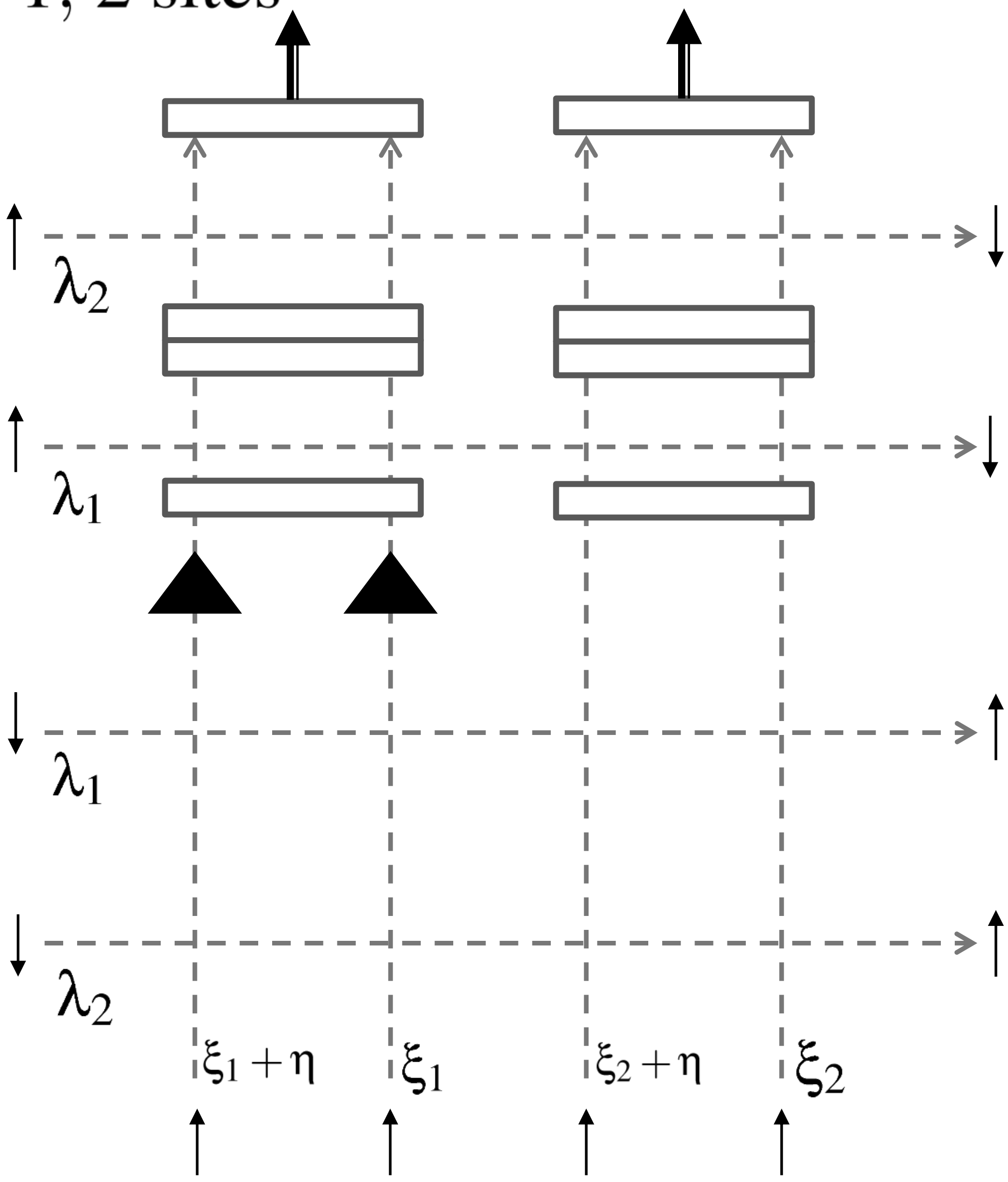
Example: $S = 1$, 2 sites



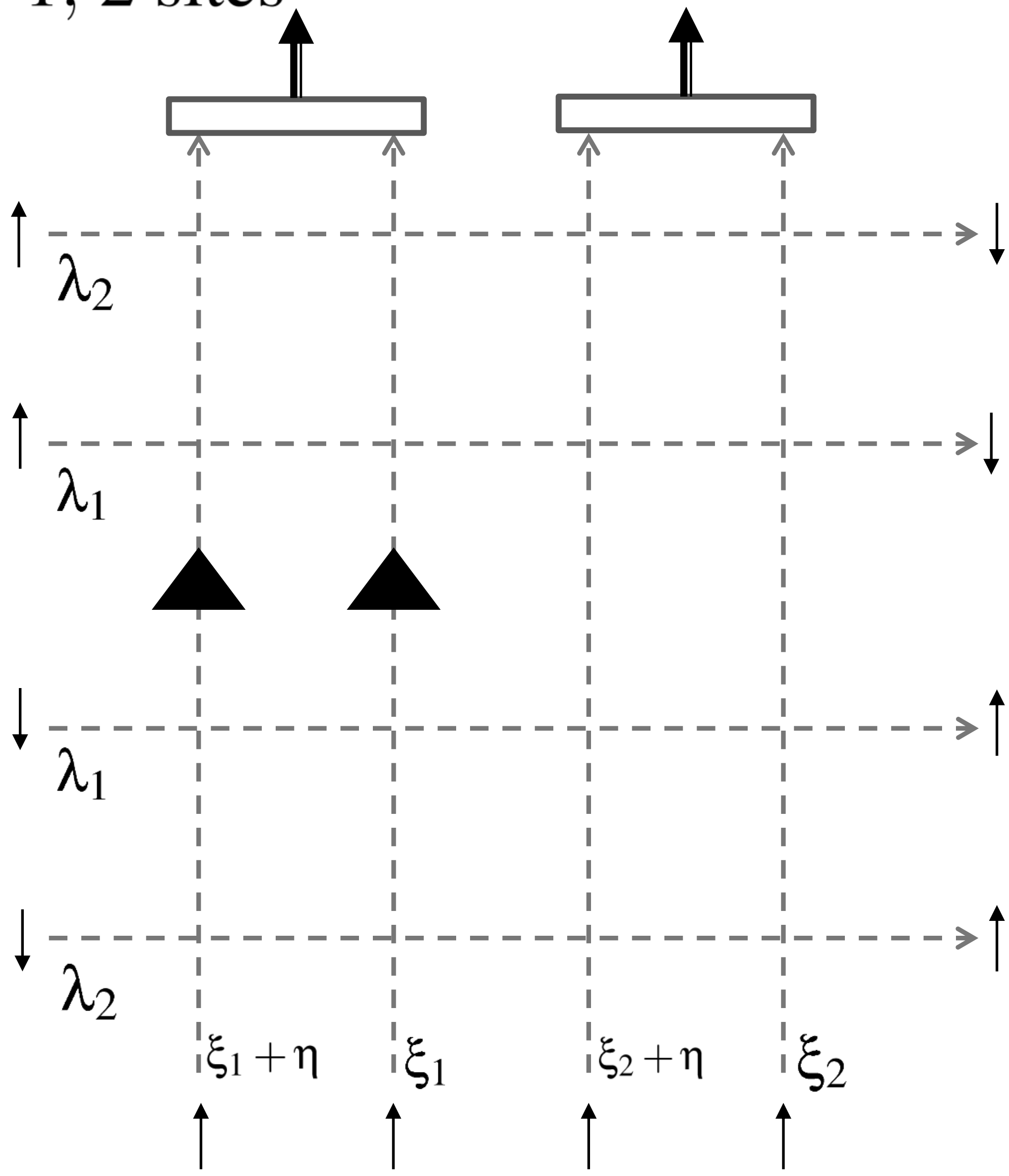
Example: $S = 1$, 2 sites



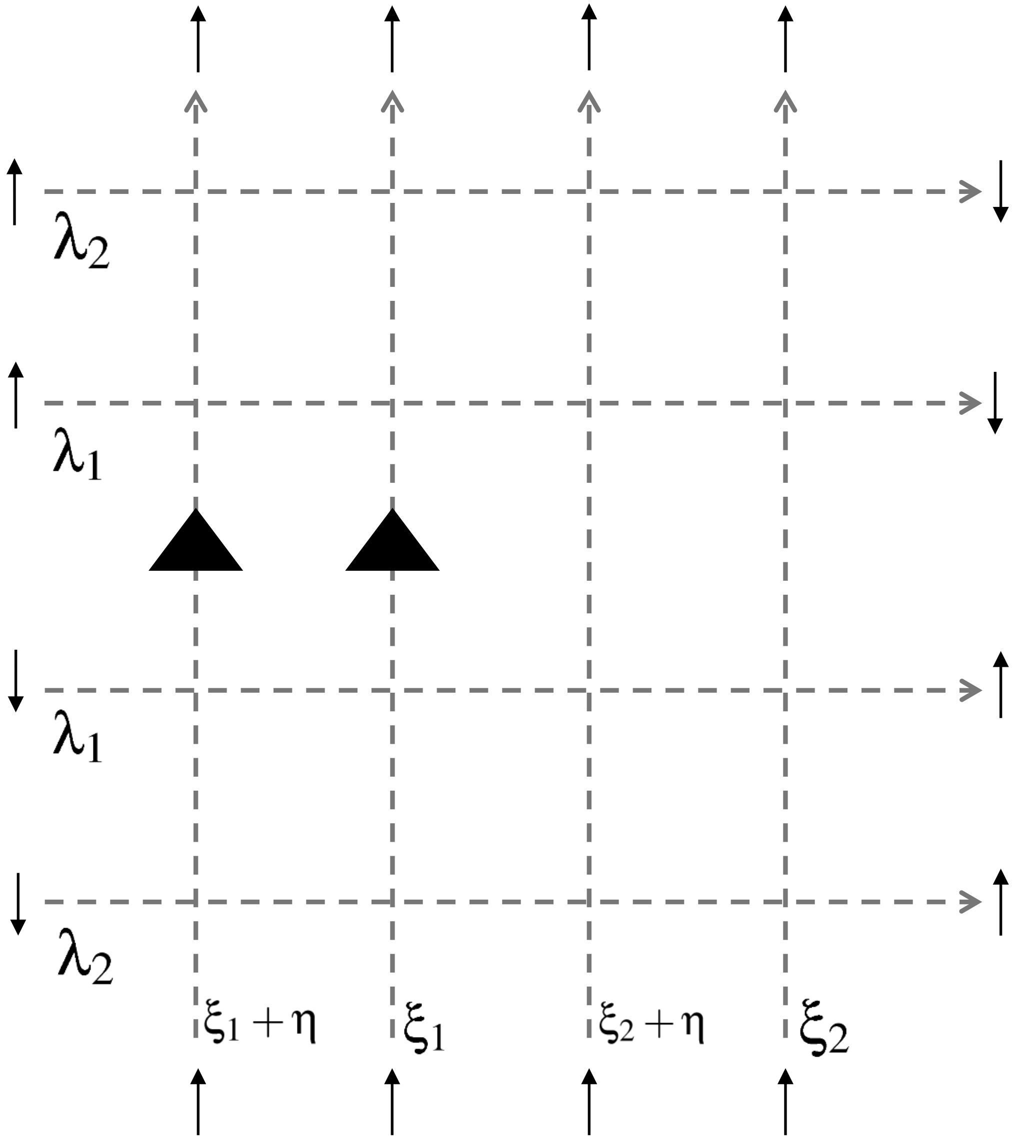
Example: $S = 1$, 2 sites



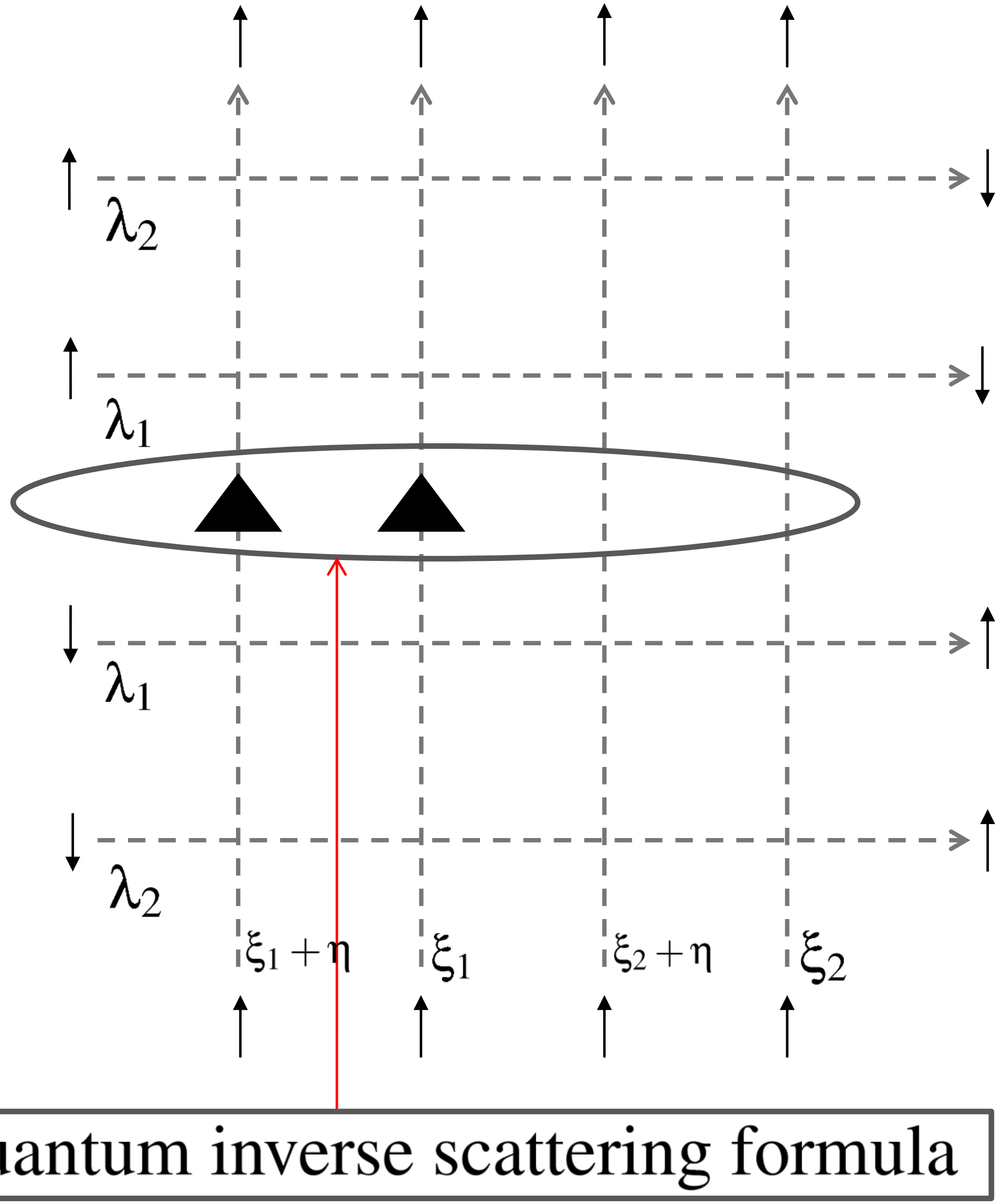
Example: $S = 1$, 2 sites



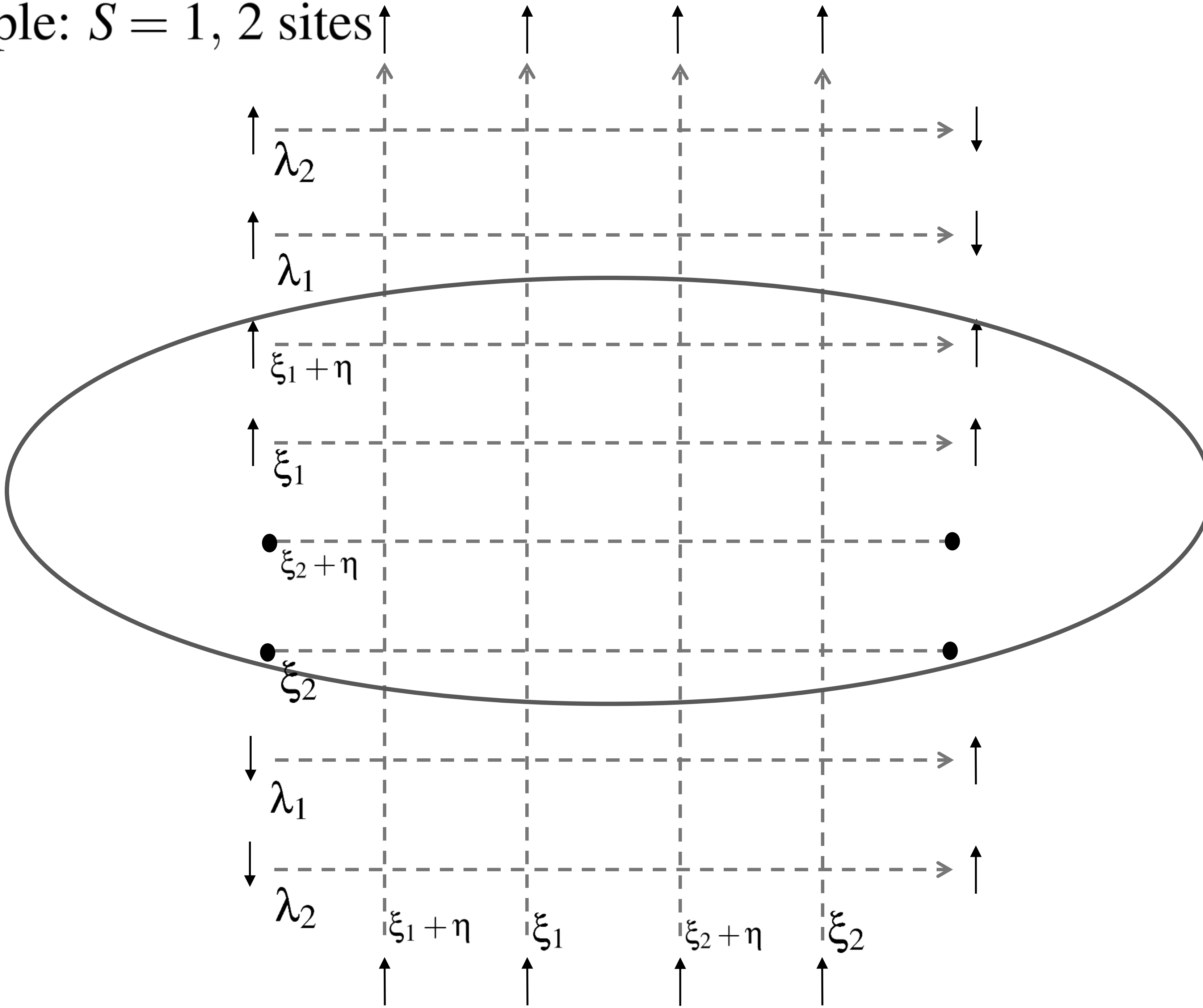
Example: $S = 1$, 2 sites



Example: $S = 1$, 2 sites



Example: $S = 1$, 2 sites



Reduction of elementary operators of higher spins to elementary operators of spin-1/2 (cf. Deguchi's talk)

$$\widehat{E}_1^{i,j\,(\ell\,w)} = \widehat{N}_{i,j}^{(\ell)}\, e^{-(i-j)\Lambda_1\,\delta(w,p)}\, P_{12\ldots\ell}^{(\ell)} \sum_{(\varepsilon_\beta(j))_\ell} \chi_{12\cdots\ell}\, e_1^{\varepsilon_1'(i),\,\varepsilon_1(j)} \cdots e_\ell^{\varepsilon_\ell'(i),\,\varepsilon_\ell(j)}\, \chi_{12\cdots\ell}^{-1}$$

$$\chi_{12\cdots L} = \Phi_1(w_1)\Phi_2(w_2)\cdots\Phi_L(w_L)$$

$$\Phi(w) = \mathrm{diag}(1, \exp(w))$$

$$P^{(\ell)} = \sum_{n=0}^{\ell} ||\ell, n\rangle\langle\ell, n||$$

$$\Lambda_1 = \xi_1 - (\ell - 1)\eta/2$$

Reduction of elementary operators of higher spins to elementary operators of spin-1/2 (cf. Deguchi's talk)

$$\widehat{E}_1^{i,j\,(\ell\, w)} = \widehat{N}_{i,j}^{(\ell)}\, e^{-(i-j)\Lambda_1\, \delta(w,p)}\, P_{12\ldots\ell}^{(\ell)} \sum_{(\varepsilon_\beta(j))_\ell} \chi_{12\cdots\ell}\, e_1^{\varepsilon_1'(i),\,\varepsilon_1(j)} \cdots e_\ell^{\varepsilon_\ell'(i),\,\varepsilon_\ell(j)}\, \chi_{12\cdots\ell}^{-1}$$

$$\chi_{12\cdots L} = \Phi_1(w_1)\Phi_2(w_2)\cdots\Phi_L(w_L)$$

$$P^{(\ell)} = \sum_{n=0}^{\ell} ||\ell, n\rangle\langle \ell, n||$$

$$\Phi(w) = \mathrm{diag}(1, \exp(w))$$

$$\Lambda_1 = \xi_1 - (\ell - 1)\eta/2$$

Quantum inverse scattering formula

Reduction of elementary operators of higher spins to elementary operators of spin-1/2 (cf. Deguchi's talk)

$$\widehat{E}_1^{i,j\,(\ell\, w)} = \widehat{N}_{i,j}^{(\ell)}\, e^{-(i-j)\Lambda_1\,\delta(w,p)}\, P_{12\ldots\ell}^{(\ell)} \sum_{(\varepsilon_\beta(j))_\ell} \chi_{12\cdots\ell}\, e_1^{\varepsilon_1'(i),\,\varepsilon_1(j)} \cdots e_\ell^{\varepsilon_\ell'(i),\,\varepsilon_\ell(j)}\, \chi_{12\cdots\ell}^{-1}$$

$$\chi_{12\cdots L} = \Phi_1(w_1)\Phi_2(w_2)\cdots\Phi_L(w_L)$$

$$\Phi(w) = \mathrm{diag}(1, \exp(w))$$

$$P^{(\ell)} = \sum_{n=0}^{\ell} ||\ell, n\rangle\langle \ell, n||$$

$$\Lambda_1 = \xi_1 - (\ell - 1)\eta/2$$

$$\widehat{E}_1^{i\,j\,(\ell\, w)} = \widehat{N}_{i,j}^{(\ell)}\, e^{-(i-j)\Lambda_1\delta(w,p)} \times$$

$$\times \quad P_{1\cdots\ell}^{(\ell)} \cdot \chi_{12\cdots\ell} \sum_{(\varepsilon_\beta(j))_\ell} T^{(1\, w)}_{\varepsilon_1(j),\,\varepsilon_1'(i)}(w_1) \cdots T^{(1\, w)}_{\varepsilon_\ell(j),\,\varepsilon_\ell'(i)}(w_\ell) \prod_{k=1}^{\ell} \left(A^{(1\,w)}(w_k) + D^{(1\,w)}(w_k)\right)^{-1} \chi_{12\cdots\ell}^{-1}$$

Reduction of elementary operators of higher spins to elementary operators of spin-1/2 (cf. Deguchi's talk)

$$\widehat{E}_1^{i,j\,(\ell\, w)} = \widehat{N}_{i,j}^{(\ell)}\, e^{-(i-j)\Lambda_1\, \delta(w,p)}\, P_{12\ldots\ell}^{(\ell)} \sum_{(\varepsilon_\beta(j))_\ell} \chi_{12\cdots\ell}\, e_1^{\varepsilon_1'(i),\, \varepsilon_1(j)} \cdots e_\ell^{\varepsilon_\ell'(i),\, \varepsilon_\ell(j)}\, \chi_{12\cdots\ell}^{-1}$$

$$\chi_{12\cdots L} = \Phi_1(w_1)\Phi_2(w_2)\cdots\Phi_L(w_L)$$

$$\Phi(w) = \mathrm{diag}(1, \exp(w))$$

$$P^{(\ell)} = \sum_{n=0}^{\ell} ||\ell, n\rangle\langle \ell, n||$$

$$\Lambda_1 = \xi_1 - (\ell-1)\eta/2$$

$$\widehat{E}_1^{i\,j\,(\ell\, w)} = \widehat{N}_{i,j}^{(\ell)}\, e^{-(i-j)\Lambda_1 \delta(w,p)} \times$$

$$\times \quad P_{1\cdots\ell}^{(\ell)} \cdot \chi_{12\cdots\ell} \sum_{(\varepsilon_\beta(j))_\ell} T_{\varepsilon_1(j),\, \varepsilon_1'(i)}^{(1\, w)}(w_1) \cdots T_{\varepsilon_\ell(j),\, \varepsilon_\ell'(i)}^{(1\, w)}(w_\ell) \prod_{k=1}^{\ell} \left(A^{(1\, w)}(w_k) + D^{(1\, w)}(w_k)\right)^{-1} \chi_{12\cdots\ell}^{-1}$$

1. Formulation of correlation functions

2. multiple sum representation

Emptiness formation probability

$$\langle \underbrace{\Uparrow\Uparrow\cdots\Uparrow}_{m} \rangle = \frac{1}{\prod_{1\le j<r\le 2s} \mathrm{sh}^m (r-j)\eta \prod_{1\le k<l\le m} \prod_{j=1}^{2s}\prod_{r=1}^{2s} \mathrm{sh}((r-j)\eta + (\xi_k - \xi_l)/i)}$$

$$\times \sum_{c_1=1}^{M} \sum_{\substack{c_2=1 \\ c_2\neq c_1}}^{M} \cdots \sum_{\substack{c_{2sm}=1 \\ c_{2sm}\neq c_1,\cdots,c_{2sm-1}}}^{M} H^{(2s)}(X_1,\cdots,X_{2sm};\{\xi_p\}) \det(\phi((c_j)_{2sm};\{\xi_p\}))$$

$$H^{(2s)}(X_1,\cdots,X_{2sm};\{\xi_p\})$$

$$= \frac{i^{ms(1-2ms)}}{\prod_{1\le l<k\le 2sm} \sin(X_{c_k} - X_{c_l} + i\eta)} \prod_{j=1}^{2sm}\prod_{b=1}^{m}\prod_{\beta=1}^{2s-1} \sin(X_{c_j} - \xi_b + i\beta\eta)$$

$$\times \prod_{l=1}^{m}\prod_{r_l=1}^{2s} \left(\prod_{k=1}^{l-1} \sin(X_{c_{2s(l-1)+r_l}} - \xi_k + 2is\eta) \prod_{k=l+1}^{m} \sin(X_{c_{2s(l-1)+r_l}} - \xi_k) \right)$$

3. Take the thermodynamic limit

properties of the ground state
2*s* string (cf. Sato's talk)

multiple integral representation

Emptiness formation probability

$$\langle \underbrace{\Uparrow\Uparrow\cdots\Uparrow}_{m} \rangle = \frac{1}{\prod_{1\le j<r\le 2s}\mathrm{sh}^m(r-j)\eta \prod_{1\le k<l\le m}\prod_{j=1}^{2s}\prod_{r=1}^{2s}\mathrm{sh}((r-j)\eta+(\xi_k-\xi_l)/i)}$$

$$\times \prod_{l=1}^{2sm}\left(\sum_{k=1}^{2s}\int_{-\pi/2+i(-k+1/2)\eta}^{\pi/2+i(-k+1/2)\eta} \mathrm{d}X_l\right) H^{(2s)}(X_1,\cdots,X_{2sm})\mathrm{det}S(X_1,\cdots,X_{2sm})$$

$$H^{(2s)}((X_l)_{2sm}) = \frac{i^{ms(1-2ms)}}{\prod_{1\le l<k\le 2sm}\sin(X_k-X_l+i\eta+\epsilon_{k,l})}$$

$$\times \prod_{l=1}^{2sm}\prod_{k=1}^{m}\prod_{p=1}^{2s-1}\sin(X_l-\xi_k+i(2s-p)\eta)$$

$$\times \prod_{l=1}^{m}\prod_{r_l=1}^{2s}\left(\prod_{k=1}^{l-1}\sin(X_{2s(l-1)+r_l}-\xi_k+2is\eta)\prod_{k=l+1}^{m}\sin(X_{2s(l-1)+r_l}-\xi_k)\right)$$

$$S_{j,2s(l-1)+k} = \begin{cases} \rho(X_j-\xi_l+i(k-1/2)\eta) & \text{if } X_j-\nu_j = i(1/2-k)\eta \\ 0 & \text{otherwise} \end{cases}$$

$$\epsilon_{k,l} = \begin{cases} i\epsilon & \text{for } \mathrm{Im}(X_k-X_l)>0 \\ -i\epsilon & \text{for } \mathrm{Im}(X_k-X_l)<0 \end{cases}$$

$$\rho(X) = \frac{1}{2\pi}\prod_{n=1}^{\infty}\left(\frac{1-q^{2n}}{1+q^{2n}}\right)^2 \frac{\theta_3(X/\pi;\tau)}{\theta_4(X/\pi;\tau)} \qquad \theta_3(v;\tau) = \sum_{n=-\infty}^{\infty} \mathrm{e}^{\pi i\tau n^2+2\pi i n v} \quad \theta_4(v;\tau) = \sum_{n=-\infty}^{\infty} \mathrm{e}^{\pi i\tau n^2+2\pi i n(v+1/2)} \qquad \tau = i\eta/\pi$$

multiple integral representation

General correlation function

$$\widehat{F}_m^{(2s\,p)}(\{i_k, j_k\}) = \langle \psi_g^{(2s\,w)} | \prod_{k=1}^{m} \widehat{E}_k^{i_k, j_k\,(2s\,w)} | \psi_g^{(2s\,w)} \rangle / \langle \psi_g^{(2s\,w)} | \psi_g^{(2s\,w)} \rangle$$

$$= \widehat{C}^{(2s)}(\{i_k, j_k\}) \left(\int_{-\pi/2+i\epsilon}^{\pi/2+i\epsilon} + \cdots + \int_{-\pi/2-i(2s-1)\eta+i\epsilon}^{\pi/2-i(2s-1)\eta+i\epsilon} \right) \mathrm{d}X_1 \cdots \left(\int_{-\pi/2+i\epsilon}^{\pi/2+i\epsilon} + \cdots + \int_{-\pi/2-i(2s-1)\eta+i\epsilon}^{\pi/2-i(2s-1)\eta+i\epsilon} \right) \mathrm{d}X_{\alpha_+}$$

$$\times \left(\int_{-\pi/2+i\epsilon}^{\pi/2+i\epsilon} + \cdots + \int_{-\pi/2-i(2s-1)\eta+i\epsilon}^{\pi/2-i(2s-1)\eta+i\epsilon} \right) \mathrm{d}X_{\alpha_+ +1} \cdots \left(\int_{-\pi/2+i\epsilon}^{\pi/2+i\epsilon} + \cdots + \int_{-\pi/2-i(2s-1)\eta+i\epsilon}^{\pi/2-i(2s-1)\eta+i\epsilon} \right) \mathrm{d}X_{2sm}$$

$$\times \sum_{\alpha^+(\{\epsilon_j\})} Q(\{\epsilon_j, \epsilon'_j\}; X_1, \cdots, X_{2sm}) \mathrm{det} S(X_1, \cdots, X_{2sm})$$

$$Q(\{\epsilon_j, \epsilon'_j\}; X_1, \cdots, X_{2sm})$$

$$= (-1)^{\alpha_+} \frac{\prod_{j \in \alpha^-} \left(\prod_{k=1}^{j-1} \sin(\widetilde{X}_j - v_k^{(2s)} + i\eta) \prod_{k=j+1}^{2sm} \sin(\widetilde{X}_j - v_k^{(2s)}) \right)}{\prod_{1 \le k < l \le 2sm} \sin(X_l - X_k + i\eta + \epsilon_{l,k})}$$

$$\times \frac{\prod_{j \in \alpha^+} \left(\prod_{k=1}^{j-1} \sin(\widetilde{X}'_j - v_k^{(2s)} - i\eta) \prod_{k=j+1}^{2sm} \sin(\widetilde{X}'_j - v_k^{(2s)}) \right)}{\prod_{1 \le k < l \le 2sm} \sin(v_k^{(2s)} - v_l^{(2s)})}.$$

$$(\widetilde{X}'_{j'_{\max}}, \cdots, \widetilde{X}'_{j'_{\min}}, X_{j_{\min}}, \cdots, X_{j_{\max}}) = (X_1, \cdots, X_{2sm})$$

$$\widehat{C}^{(2s)}(\{i_k, j_k\}) = \prod_{k=1}^{m} \sqrt{ \left[\begin{array}{c} 2s \\ i_k \end{array} \right]_q \left[\begin{array}{c} 2s \\ j_k \end{array} \right]_q^{-1} }$$

Example: $S = 1$

$$\langle \Uparrow \rangle = \frac{1}{4\pi^2} \prod_{n=1}^{\infty} \left(\frac{1-q^{2n}}{1+q^{2n}} \right)^4$$

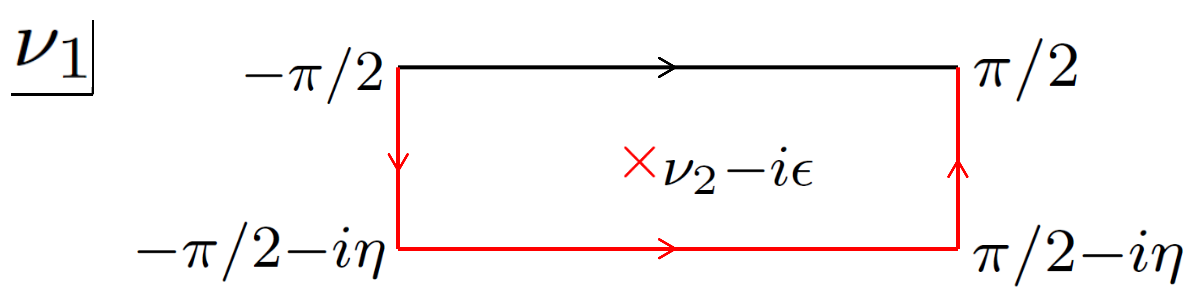
$$\times \left\{ \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_1 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_1 + i\eta/2)\sin(\nu_2 - i\eta/2)}{\sin i\eta \sin(\nu_2 - \nu_1 - i\epsilon)} \frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)} \right.$$

$$\left. - \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_1 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_1 - i\eta/2)\sin(\nu_2 + i\eta/2)}{\sin i\eta \sin(\nu_2 - \nu_1 + 2i\eta)} \frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)} \right\}.$$

Example: $S = 1$

$$
\begin{aligned}
&\langle \Uparrow \rangle \\
&= \frac{1}{4\pi^2} \prod_{n=1}^{\infty} \left(\frac{1 - q^{2n}}{1 + q^{2n}} \right)^4 \\
&\times \left\{ \boxed{\int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_1 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_1 + i\eta/2)\sin(\nu_2 - i\eta/2)}{\sin i\eta \sin(\nu_2 - \nu_1 - i\epsilon)} \frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)}} \right. \\
&\left. - \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_1 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_1 - i\eta/2)\sin(\nu_2 + i\eta/2)}{\sin i\eta \sin(\nu_2 - \nu_1 + 2i\eta)} \frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)} \right\}.
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
&\int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_1 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_1 - i\eta/2)\sin(\nu_2 - i\eta/2)}{\sin i\eta \sin(\nu_1 - \nu_2 - i\eta)} \frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)} \\
&+ 2\pi i \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_2 - i\eta/2)\sin(\nu_2 + i\eta/2)}{\sin i\eta} \frac{\theta_3^2(\nu_2/\pi;\tau)}{\theta_4^2(\nu_2/\pi;\tau)}.
\end{aligned}
}
$$



Example: $S = 1$

$$\langle \Uparrow \rangle = \frac{1}{4\pi^2}\prod_{n=1}^{\infty}\left(\frac{1-q^{2n}}{1+q^{2n}}\right)^4$$

$$\times\left\{\int_{-\pi/2}^{\pi/2}\mathrm{d}\nu_1\int_{-\pi/2}^{\pi/2}\mathrm{d}\nu_2\frac{\sin(\nu_1+i\eta/2)\sin(\nu_2-i\eta/2)}{\sin i\eta\sin(\nu_2-\nu_1-i\epsilon)}\frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)}\right.$$

$$\left.-\int_{-\pi/2}^{\pi/2}\mathrm{d}\nu_1\int_{-\pi/2}^{\pi/2}\mathrm{d}\nu_2\frac{\sin(\nu_1-i\eta/2)\sin(\nu_2+i\eta/2)}{\sin i\eta\sin(\nu_2-\nu_1+2i\eta)}\frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)}\right\}.$$

$$\int_{-\pi/2}^{\pi/2}\mathrm{d}\nu_1\int_{-\pi/2}^{\pi/2}\mathrm{d}\nu_2\frac{\sin(\nu_1-i\eta/2)\sin(\nu_2-i\eta/2)}{\sin i\eta\sin(\nu_1-\nu_2-i\eta)}\frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)}$$
$$+2\pi i\int_{-\pi/2}^{\pi/2}\mathrm{d}\nu_2\frac{\sin(\nu_2-i\eta/2)\sin(\nu_2+i\eta/2)}{\sin i\eta}\frac{\theta_3^2(\nu_2/\pi;\tau)}{\theta_4^2(\nu_2/\pi;\tau)}.$$

$$\int_{-\pi/2}^{\pi/2}\mathrm{d}\nu_1\int_{-\pi/2}^{\pi/2}\mathrm{d}\nu_2\frac{\sin(\nu_1-i\eta/2)\sin(\nu_2-i\eta/2)}{\sin i\eta\sin(\nu_1-\nu_2-i\eta)}\frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)}$$

Example: $S = 1$

$$\langle \Uparrow \rangle = \frac{1}{4\pi^2} \prod_{n=1}^{\infty} \left(\frac{1-q^{2n}}{1+q^{2n}} \right)^4$$

$$\times \left\{ \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_1 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_1 + i\eta/2)\sin(\nu_2 - i\eta/2)}{\sin i\eta \sin(\nu_2 - \nu_1 - i\epsilon)} \frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)} \right.$$

$$\left. - \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_1 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_1 - i\eta/2)\sin(\nu_2 + i\eta/2)}{\sin i\eta \sin(\nu_2 - \nu_1 + 2i\eta)} \frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)} \right\}.$$

$$\int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_1 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_1 - i\eta/2)\sin(\nu_2 - i\eta/2)}{\sin i\eta \sin(\nu_1 - \nu_2 - i\eta)} \frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)}$$

$$+ 2\pi i \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_2 - i\eta/2)\sin(\nu_2 + i\eta/2)}{\sin i\eta} \frac{\theta_3^2(\nu_2/\pi;\tau)}{\theta_4^2(\nu_2/\pi;\tau)}.$$

$$\int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_1 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_1 - i\eta/2)\sin(\nu_2 - i\eta/2)}{\sin i\eta \sin(\nu_1 - \nu_2 - i\eta)} \frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)}$$

Example: $S = 1$

$$
\langle \Uparrow \rangle = \frac{1}{4\pi^2} \prod_{n=1}^{\infty} \left(\frac{1-q^{2n}}{1+q^{2n}} \right)^4
$$

$$
\times \left\{ \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_1 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_1 + i\eta/2)\sin(\nu_2 - i\eta/2)}{\sin i\eta \sin(\nu_2 - \nu_1 - i\epsilon)} \frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)} \right.
$$

$$
\left. - \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_1 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_1 - i\eta/2)\sin(\nu_2 + i\eta/2)}{\sin i\eta \sin(\nu_2 - \nu_1 + 2i\eta)} \frac{\theta_3(\nu_1/\pi;\tau)\theta_3(\nu_2/\pi;\tau)}{\theta_4(\nu_1/\pi;\tau)\theta_4(\nu_2/\pi;\tau)} \right\}.
$$

$\downarrow$

$$
\boxed{\langle \Uparrow \rangle = \frac{1}{2\pi \mathrm{sh}\eta} \prod_{n=1}^{\infty} \left(\frac{1-q^{2n}}{1+q^{2n}} \right)^4 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_2 - i\eta/2)\sin(\nu_2 + i\eta/2)\theta_3^2(\nu_2/\pi;\tau)}{\theta_4^2(\nu_2/\pi;\tau)}}
$$

Example: $S = 1$

$$\left\langle \Uparrow \right\rangle = \frac{1}{2\pi \mathrm{sh}\eta} \prod_{n=1}^{\infty} \left(\frac{1 - q^{2n}}{1 + q^{2n}} \right)^4 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_2 - i\eta/2) \sin(\nu_2 + i\eta/2) \theta_3^2(\nu_2/\pi;\tau)}{\theta_4^2(\nu_2/\pi;\tau)}$$

Example: $S = 1$

$$\langle \Uparrow \rangle = \frac{1}{2\pi \mathrm{sh}\eta} \prod_{n=1}^{\infty} \left(\frac{1-q^{2n}}{1+q^{2n}} \right)^4 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_2 - i\eta/2)\sin(\nu_2 + i\eta/2)\theta_3^2(\nu_2/\pi;\tau)}{\theta_4^2(\nu_2/\pi;\tau)}$$

$$\int_{-\pi/2}^{\pi/2} \mathrm{d}x \frac{\cos 2x\, \theta_3^2(x/\pi;\tau)}{\theta_4^2(x/\pi;\tau)} = \frac{4\pi \mathrm{sh}\eta}{\mathrm{ch}2\eta - 1} \left(\frac{1+q^{2n}}{1-q^{2n}} \right)^4$$

$$\langle \Uparrow \rangle = \frac{\mathrm{ch}\eta}{4\pi \mathrm{sh}\eta} \prod_{n=1}^{\infty} \left(\frac{1-q^{2n}}{1+q^{2n}} \right)^4 \int_{-\pi/2}^{\pi/2} \mathrm{d}x \frac{\theta_3^2(x/\pi;\tau)}{\theta_4^2(x/\pi;\tau)} - \frac{1}{\mathrm{ch}2\eta - 1}$$

Example: $S = 1$

$$\langle \uparrow \rangle = \frac{1}{2\pi \mathrm{sh}\eta} \prod_{n=1}^{\infty} \left(\frac{1 - q^{2n}}{1 + q^{2n}} \right)^4 \int_{-\pi/2}^{\pi/2} \mathrm{d}\nu_2 \frac{\sin(\nu_2 - i\eta/2)\sin(\nu_2 + i\eta/2)\theta_3^2(\nu_2/\pi;\tau)}{\theta_4^2(\nu_2/\pi;\tau)}$$

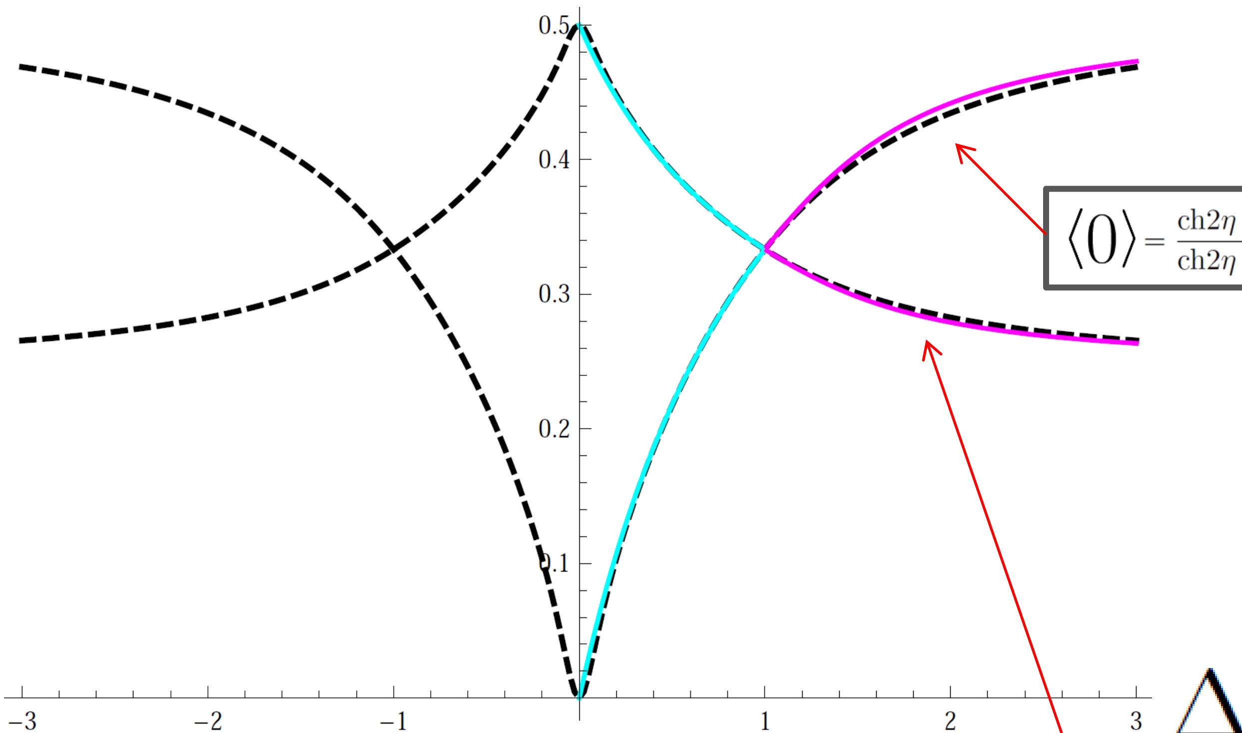
$$\int_{-\pi/2}^{\pi/2} \mathrm{d}x \frac{\cos 2x \theta_3^2(x/\pi;\tau)}{\theta_4^2(x/\pi;\tau)} = \frac{4\pi \mathrm{sh}\eta}{\mathrm{ch}2\eta - 1} \left(\frac{1 + q^{2n}}{1 - q^{2n}} \right)^4$$

$$\langle \uparrow \rangle = \frac{\mathrm{ch}\eta}{4\pi \mathrm{sh}\eta} \prod_{n=1}^{\infty} \left(\frac{1 - q^{2n}}{1 + q^{2n}} \right)^4 \int_{-\pi/2}^{\pi/2} \mathrm{d}x \frac{\theta_3^2(x/\pi;\tau)}{\theta_4^2(x/\pi;\tau)} - \frac{1}{\mathrm{ch}2\eta - 1}$$

$$\prod_{n=1}^{\infty} \left(\frac{1 - q^{2n}}{1 + q^{2n}} \right)^2 \frac{\theta_3(x/\pi;\tau)}{\theta_4(x/\pi;\tau)} = \sum_{n=-\infty}^{\infty} \frac{e^{2ixn}}{\mathrm{ch}n\eta}$$

$$\langle \uparrow \rangle = \frac{\mathrm{ch}\eta}{4 \mathrm{sh}\eta} \sum_{n=-\infty}^{\infty} \frac{1}{\mathrm{ch}^2 n\eta} - \frac{1}{\mathrm{ch}2\eta - 1}$$

comparison with exact diagonalization



$$\langle 0 \rangle = \frac{\mathrm{ch}2\eta+1}{\mathrm{ch}2\eta-1} - \frac{\mathrm{ch}\eta}{2\pi\mathrm{sh}\eta}\left(\frac{1-q^{2n}}{1+q^{2n}}\right)^4 \int_{-\pi/2}^{\pi/2} \mathrm{d}x \frac{\theta_3^2(x/\pi;\tau)}{\theta_4^2(x/\pi;\tau)}$$

$$\langle \Uparrow \rangle = \langle \Downarrow \rangle = \frac{\mathrm{ch}\eta}{4\mathrm{sh}\eta} \sum_{n=-\infty}^{\infty} \frac{1}{\mathrm{ch}^2 n\eta} - \frac{1}{\mathrm{ch}2\eta - 1}$$

XXX limit

$$\langle \Uparrow \rangle = \frac{\mathrm{ch}\eta}{4\mathrm{sh}\eta} \sum_{n=-\infty}^{\infty} \frac{1}{\mathrm{ch}^2 n\eta} - \frac{1}{\mathrm{ch}2\eta - 1}$$

$$\sum_{n=-\infty}^{\infty} \frac{1}{\mathrm{ch}^2 n\eta} \to \frac{2}{\eta}$$

$$\lim_{\eta\to 0} \langle \Uparrow \rangle = \lim_{\eta\to 0} \frac{\mathrm{ch}\eta(\mathrm{ch}2\eta - 1) - 2\eta\mathrm{sh}\eta}{2\eta\mathrm{sh}\eta(\mathrm{ch}2\eta - 1)} = \frac{1}{3}$$

This talk

- multiple integral representation for correlation functions of higher spin XXZ chain in the massive regime
- one point functions

Future problems

- spontaneous magnetization
- asymptotics